Code No: 151AA **R18**

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD
B. Tech I Year I Semester Examinations, January/February - 2024
MATHEMATICS - I
(Common to CE, EEE, ME, ECE, CSE, EIE, IT, MCT, MMT, ECM, AE, MIE, PTM, CSBS, CSIT, ITE, CE(SE), CSE(CS), CSE(AI&ML), CSE(DS), CSE(IOT), CSE(N), AI&DS, AI&ML, CSD)

Time: 3 Hours **Max. Marks: 75**

Note: i) Question paper consists of Part A, Part B.
ii) Part A is compulsory, which carries 25 marks. In Part A, answer all questions.
iii) In Part B, Answer any one question from each unit. Each question carries 10 marks and may have a, b as sub questions.

PART – A

(25 Marks)

1.a) Find k such that the matrix $A = \begin{bmatrix} 1 & 1 & -2 \\ 3 & -1 & 1 \\ k & 3 & -6 \end{bmatrix}$ is singular? [2]

b) Write down two properties of orthogonal matrix? [3]

c) Define Rank, Index and Signature of a matrix. [2]

d) Two of the Eigen values of $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ are 3 and 6. Find the Eigen values of A^{-1}. [3]

e) Define Divergent sequence with an example. [2]

f) Define Cauchy's root test. [3]

g) State Lagrange's mean value theorem. [2]

h) Define Beta function. [3]

i) Define Continuity. [2]

j) Find $\frac{\partial u}{\partial t}, if\ u = tan^{-1}\left(\frac{y}{x}\right)$ and $x = e^t - e^{-t}$ and $y = e^t + e^{-t}$. [3]

PART - B

(50 Marks)

2.a) Reduce the matrix $\begin{bmatrix} 1 & 4 & 5 & 2 \\ 2 & 8 & 6 & 7 \\ 3 & 5 & 2 & 1 \\ -1 & 2 & 3 & 0 \end{bmatrix}$ to echelon form and find its Rank

b) Show that the system of equations is consistent and solve it.
$2x - y - z = 2; x + 2y + z = 2; 4x - 7y - 5z = 2$ [4+6]

OR

3. Solve the system of equations $10x + y + z = 12$;
$2x + 10y + z = 13;$
$2x + 2y + 10z = 14$ by Gauss seidal method. [10]

4. Reduce the quadratic form $2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2zx$ to canonical form by orthogonal transformation and hence find the rank, index, signature and nature of the quadratic form. [10]

OR

5. Verify Cayley-Hamilton theorem and find the inverse of the matrix $A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$. [10]

6. Discuss the convergence of the series $\sum \frac{4.7.10.....(3n+1)}{1.2.3........n} x^n$. [10]

OR

7.a) Examine whether the series $-1 + \frac{1}{2^2} - \frac{1}{3^2} + \frac{1}{4^2} - \frac{1}{5^2} + \cdots$ is absolutely convergent or conditionally convergent.

b) By Leibnitz's test, verify the series $\frac{1}{1.2} - \frac{1}{3.4} + \frac{1}{5.6} - \frac{1}{7.8} \ldots \ldots$ is convergent. [5+5]

8.a) Verify Rolle's theorem for $f(x) = e^x(sinx - cosx)$ in $\left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$.

b) Express the polynomial $2x^3 + 7x^2 + x - 6$ in powers of $(x - 2)$ using Taylor's series. [5+5]

OR

9. Prove that $\beta(m,n) = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}} dx = \int_0^1 \frac{x^{m-1}}{(1+x)^{m+n}} dx$. [10]

10.a) If $u = tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$. Find $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}$ using Euler's theorem.

b) Find $\frac{dy}{dx}$ $if\ x^y = y^x$. [5+5]

OR

11. Find the dimensions of the rectangular parallelepiped box open at the top of the maximum capacity whose surface area is 108 sq.inches. [10]

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